Numerical study of forced convection in a 3D flow of a non-Newtonian fluid through a porous duct

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Abstract

Purpose – To investigate the forced convection heat transfer to hydrodynamically and thermally fully developed laminar steady flow of power-law non-Newtonian fluid in a partially porous square duct.

Design/methodology/approach – The modified Brinkmann-Forchheimer extended Darcy model for power-law fluids is used in the porous layer. The solutions for the velocity and temperature fields are obtained numerically using the finite volume method. Computations are performed over a range of Darcy number, power-law indices, porous insert thickness and thermal conductivity ratio.

Findings – The average Nusselt number and the Fanning factor, so obtained are found to be in good agreement with the literature. It is highlighted that a heat transfer improvement is obtained when the channel is entirely porous and this enhancement is maximized at low permeability. While depending on the working conditions, heat transfer enhancement can also be obtained by filling partially the duct with the porous insert, even if the conductivity ratio is equal to 1. The results indicate also that the conductivity ratio has a strong impact on the heat transfer enhancement at high permeability, while this impact is significant beyond a critical thickness of the porous layer at low permeability. It is found that both shear-thinning ($n < 1$) and shear-thickening ($n > 1$) fluids allow obtaining the highest Nusselt number according to the properties of the porous insert. The presence of the porous insert causes a significant increase in pressure drop. This added pressure drop is found to be more important with shear thickening fluids ($n > 1$).

Research limitations/implications – The results of this paper are valid for square ducts and H1 thermal boundary condition, corresponding to an axially uniform heat flux and peripherally uniform temperature. The inertial effects are neglected in the porous region.

Practical implications – The obtained results can be used in the design of heat exchangers and in the cooling of electronic equipments.

Originality/value – This work investigates some interesting ways to enhance heat transfer in three-dimensional square ducts by using porous substrates and non-Newtonian fluids. It is believed that the case studied in this paper has not previously been investigated.

Keywords Convection, Porous materials, Newton method

Paper type Research paper

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Nomenclature

a	= height of the rectangular duct
A	= constant defined in equation (12)
b	= width of the rectangular duct
c	= specific heat
C	= inertial factor
C_t	= turtuosity factor
Da	= modified Darcy number
D_h	= hydraulic diameter
e	= dimensionless porous layer thickness

f	= Fanning factor	U	= dimensionless axial velocity
h	= peripherally averaged heat transfer coefficient	u_m	= mean velocity
k	= thermal conductivity	U_m	= dimensionless mean axial velocity
K	= permeability	x, y, z	= coordinate system
K^*	= modified permeability	X, Y, Y	= dimensionless coordinate system
n	= power-law index	*Greek*	
n'	= modified power-law index	α	= thermal diffusivity
Nu	= peripherally averaged Nusselt number	ε	= porosity
		Γ	= dimensionless viscosity
R_k	= conductivity ratio	θ	= dimensionless temperature
p	= pressure	θ_m	= dimensionless bulk temperature
P	= dimensionless pressure	λ	= binary variable
r	= aspect ratio of the duct ($= a/b$)	μ^*	= consistency factor
Re	= Reynolds number	ρ	= thermal diffusivity porosity
T	= temperature		
T_m	= bulk temperature	*Subscripts*	
T_w	= wall temperature	e	= effective
u	= axial velocity	f	= fluid

Introduction

During the last few years, an increasing interest has been devoted to fundamental studies of forced convection in channels or ducts fully or partially filled with a porous material. This is due to the wide applications of porous media in numerous engineering fields such as ceramic processing, filtration, geothermal systems, enhanced oil recovery, compact heat exchangers, and packed bed chemical reactors, etc. In the majority of these studies, the considered fluid is Newtonian. However, in industrial applications the fluid drifts away from the Newtonian behavior. For this reason, the number of studies referring to non-Newtonian fluids is in increase in order to take into account the rheological aspect.

It has been showed that the insertion of porous matters can enhance significantly the heat transfer. Huang and Vafai (1994a, b) presented an innovative approach in altering and controlling the heat transfer and frictional characteristics of an external surface. They considered an external surface on which porous cavities and porous blocks were mounted alternately. In the second, only intermittently porous cavities were used with the same flow configuration. The solutions of the problems were obtained numerically using a finite difference method. It was shown that depending on the physical conditions heat transfer enhancement can be obtained with lower pumping requirements. The results reported by the authors can be extended to various industrial applications such as chipset electronic cooling and heat exchanger design. Hadim (1994) numerically analyzed laminar forced convection in fully and partially porous parallel plate channels with discrete heat sources. It was found that when the width of the heat source and the spacing between the porous inserts are of the same order of magnitude as the channel height, the heat transfer enhancement is almost the same as in the fully porous channel while the pressure drop is significantly lower. Huang and Vafai (1994c) showed that a significant heat transfer increase could be obtained by adding porous blocks on the bottom wall of an isothermal parallel plates channel using a vorticity stream function formulation. Sung *et al.* (1995) studied numerically the effects of the height and the permeability of the porous matrix on the flow and heat transfer characteristics of forced convection in a partially porous channel. Chiem and Zhao (2004) have analyzed numerically the same problem reported by Huang and Vafai (1994c). The steady and unsteady cases were considered in their study. The situation where the porous insert is in the flow channel was studied by Tong *et al.* (1993). They showed that heat transfer could be increased. Moreover, either the partially or completely porous channel according to the choice of the physical conditions specific to each configuration, allows obtaining a maximum heat transfer. Chikh *et al.* (1995a) derived an analytical solution of non-Darcian forced convection in a partially annular porous duct. The fully developed case was considered, and the porous insert was positioned at the inner cylinder. The results obtained show that there exists a critical thickness of the porous layer at which heat transfer is minimum. In a similar configuration, the inertial effects on the flow and heat transfer were investigated by Chikh *et al.* (1995b). Both constant wall heat flux and constant wall temperature boundary conditions at the inner cylinder were investigated.

It is well known that some fluids encountered in various industrial applications do not adhere to the Newtonian model. Ai and Vafai (2005) numerically analyzed the effects of the rheological behavior of non-Newtonian fluids on the Stokes second problem. For this purpose eight models of non-Newtonian fluids were considered. For the power law model, two correlations, which give the velocity and the time required to reach the steady periodic flow, were established. Moreover, to simulate the blood behavior at unsteady state, three non-Newtonian models were adopted. It was shown that the field of the velocity and the wall shear stress is consistent across all models; however the reference shear rate and the used model affect the magnitude significantly. In the majority of the studies devoted to non-Newtonian fluids, external flows were the most analyzed in presence of a porous material. Among them are the studies of Nakayama and Shenoy (1992), Gorla and Kumari (1998) and Pop and Nakayama (1994). Chen and Hadim (1998) numerically studied laminar forced convection in a packed bed saturated with a power-law fluid. Their results indicated that shear thinning fluid provide a higher heat transfer and a lower pressure drop than Newtonian fluids in porous media.

The above literature review revealed that most of the undertaken works were devoted to two-dimensional configurations. Three-dimensional studies of forced convection are rather limited, although in numerous engineering applications, three-dimensional effects cannot be neglected. Kuzai *et al.* (1991) experimentally investigated the heat transfer enhancement in a metal-wool-filled square duct; they considered high Reynolds number in low permeability fibrous media. Hwang and Wu (1995) reported on drag and heat transfer measurements and numerical analysis for a square packed-sphere channel, the model used did not take into account the effects of variable porosity. Recently, Chen and Hadim (1999) performed a numerical investigation of three-dimensional non-Darcy forced convection in a square porous duct. They took into account the thermal dispersion and the variation of the porosity near the duct walls. In their results, reported for three different thermal boundary conditions, it was shown that the channeling phenomenon and the thermal dispersion effects were reduced considerably in a three-dimensional duct compared with previously reported results for a two-dimensional channel. The use of a non-Newtonian fluid for heat transfer improvement in rectangular ducts was proposed by numerous authors as reviewed by Hartnett and Kostic (1989). However, in these investigations, the ducts were not filled with a porous medium. The literature survey has revealed that a partially porous channel is a more attractive heat transfer enhancement technique because of its reduced pressure drop. It was also shown that with the correct choice of the governing parameters, a significant heat transfer improvement could be obtained. It appears that this technique was not extended to rectangular ducts especially when the working fluid is non-Newtonian.

In the present work, a numerical investigation of steady laminar forced convection flow in a three dimensional square duct partially or completely filled with a porous medium and saturated with a power-law fluid was performed. Heat transfer and hydrodynamic results were reported for the configuration in which the channel walls were subjected to H1 thermal boundary condition. Parametric studies were conducted to examine the effects of the Darcy number, the power law index, and the thickness of the porous material on the heat transfer as well as on the flow field in the duct.

Mathematical formulation

The duct configuration and coordinates system are depicted in Figure 1. It is assumed that the flow in the duct is steady, incompressible, hydrodynamically and thermally developed. The flow in the porous medium is modeled using a modified Brinkman-Forchheimer-extended Darcy model for power-law fluids reported by Shenoy (1994), which takes into account boundary and inertia effects. The porous medium is considered to be isotropic homogeneous, saturated with a non-Newtonian fluid, which obeys to the power-law model, and the local thermal equilibrium is assumed to prevail between the fluid and the solid matrix. The thermophysical properties of the solid matrix and the fluid are considered to be constant except for the viscosity of the power-law fluid, which is a function of the shear rate while the viscous dissipation is neglected. Thus, the flow is unidirectional and it is expressed in terms of the axial velocity u, which does not depend on the axial position x.

The equations of continuity, momentum and energy that govern the fluid flow and heat transfer in the present study are as follows:

Continuity:

$$\frac{\partial u}{\partial x} = 0 \tag{1}$$

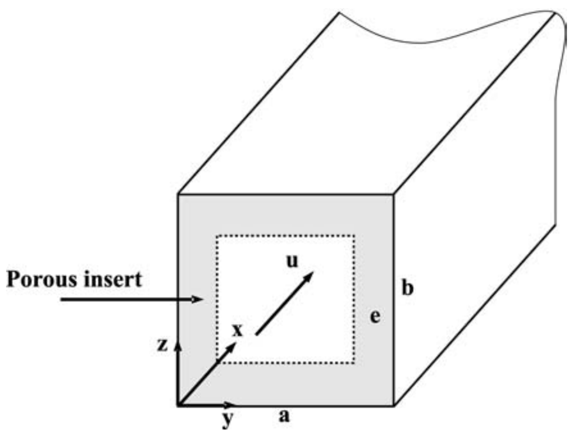

Figure 1. Physical configuration and coordinates system

Momentum

In the porous region:

$$\frac{1}{\varepsilon^{n}}\left\{\frac{\partial}{\partial y}\left[\eta\frac{\partial u}{\partial y}\right]+\frac{\partial}{\partial z}\left[\eta\frac{\partial u}{\partial z}\right]\right\}-\frac{\partial p}{\partial x}-\left(\frac{\mu^{*}}{K^{*}}|u|^{(n-1)}-\frac{\rho C}{\sqrt{K}}|u|\right)u=0 \tag{2}$$

In the fluid region:

$$\left\{\frac{\partial}{\partial y}\left[\eta\frac{\partial u}{\partial y}\right]+\frac{\partial}{\partial z}\left[\eta\frac{\partial u}{\partial z}\right]\right\}-\frac{\partial p}{\partial x}=0 \tag{3}$$

Energy

In the porous region:

$$u\frac{\partial T}{\partial x}=\frac{k_e}{\rho_f c_f}\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{4}$$

In the fluid region:

$$u\frac{\partial T}{\partial x}=\frac{k_f}{\rho_f c_f}\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{5}$$

In the above equations, y and z are the transversal coordinates, x is the axial coordinate, u is the velocity component in the axial direction, $\partial p/\partial x$ represents the axial pressure gradient that is constant under the assumed conditions, T is the temperature, and η is the viscosity. For a power-law fluid, the expression for the viscosity is given by

$$\eta=\mu^{*}\left\{\left(\frac{\partial u}{\partial y}\right)^{2}+\left(\frac{\partial u}{\partial z}\right)^{2}\right\}^{(n-1)/2} \tag{6}$$

with μ^{*} being the consistency factor and n the power-law index.

In the momentum equation, K^{*} denotes the modified permeability (Shenoy, 1994) that depends on the structure of the porous medium and the power-law index of the fluid and is given by

$$K^{*}=\frac{1}{2\cdot\mathrm{Ct}}\left(\frac{n\varepsilon}{3n+1}\right)^{n}\left(\frac{50K}{3\varepsilon}\right)^{(n+1)/2} \tag{7}$$

where K and ε are, respectively, the intrinsic permeability and the porosity of the porous medium. The turtuosity factor Ct is defined in many different ways in the literature. In this study, the expression given by Dharmadhikari and Kale (1985) is adopted:

$$\mathrm{Ct}=\frac{2}{3}\left(\frac{8n'}{9n'+3}\right)^{n'}\left(\frac{10n'-3}{6n'+1}\right)\left(\frac{75}{16}\right)^{3(10n'-3)/(10n'+11)} \tag{8}$$

and:

$$n'=n+0.3(1-n) \tag{9}$$

As a result of the hydrodynamically and thermally fully developed flow, the term $\partial T/\partial z$ in the energy equation can be expressed as:

$$\frac{\partial T}{\partial x} = \frac{\mathrm{d}T_m}{\mathrm{d}x} \tag{10}$$

The boundary conditions for the present configuration are such that a no-slip condition occurs at the impermeable walls that are considered to be subject to the H1 thermal boundary condition. The governing equations are put in dimensionless form by adopting the following non-dimensional variables:

$$X = \frac{x}{D_h}, \quad Y = \frac{y}{D_h}$$

$$U = \frac{u}{u_m}, \quad P = \frac{p}{\rho u_m^2}, \quad \theta = \frac{\alpha_f (T - T_w)}{u_m D_\mathrm{h}^2 \frac{\mathrm{d}T_m}{\mathrm{d}x}}$$

Introducing a binary variable allows using a single set of governing equations for both the fluid and porous regions. With this formulation, the interface matching conditions are satisfactorily dealt with as shown by Patankar (1980); thus, the numerical solution procedure is greatly simplified. Consequently, the equations are rewritten as follows:

Continuity:

$$\frac{\partial U}{\partial X} = 0 \tag{11}$$

Momentum:

$$\begin{aligned} &\chi\left\{\frac{\partial}{\partial Y}\left[\Gamma\frac{\partial U}{\partial Y}\right] + \frac{\partial}{\partial Z}\left[\Gamma\frac{\partial U}{\partial Z}\right]\right\} \\ &\quad - \left(\frac{r+1}{2r}\right)^2\left\{2f\,Re - \lambda\left(\frac{|U|^{n-1}}{Da^{(n+1)/2}} - A^{1/(n+1)}C\,\frac{Re}{\sqrt{Da}}|U|\right)U\right\} = 0 \end{aligned} \tag{12}$$

with:

$$\chi = \left(\lambda\left(\frac{1}{\varepsilon^n} - 1\right) + 1\right)$$

Energy:

$$(\lambda(R_\mathrm{k} - 1) + 1)\left\{\frac{\partial^2\theta}{\partial Y^2} + \frac{\partial^2\theta}{\partial Z^2}\right\} - \left(\frac{r+1}{2r}\right)^2\frac{U}{U_m} = 0 \tag{13}$$

In the above equations, *Da* and *Re* are, respectively, the modified Darcy and the generalized Reynolds numbers.

$$Re = \frac{\rho u_m^{2-n} D_\mathrm{h}^n}{\mu^*} \tag{14}$$

$$Da = \frac{(K^*)^{2/(n+1)}}{D_\mathrm{h}^2} \tag{15}$$

Since the geometry considered in this work has two symmetry planes, the governing equations are solved only in one quarter of the physical field to reduce the computations cost.

Then, the relevant dimensionless boundary conditions are:

On the duct walls:

$$U = 0, \quad \theta = 0 \tag{16a}$$

And on the symmetry planes $X = 0.5$ and $Y = 0.5$:

$$\frac{\partial U}{\partial Y} = 0, \quad \frac{\partial \theta}{\partial Z} = 0 \tag{16b}$$

$$\frac{\partial U}{\partial Z} = 0, \quad \frac{\partial \theta}{\partial Z} = 0 \tag{16c}$$

An additional constraint, which is used to deduce the axial pressure gradient, is that global mass conservation had to be satisfied. This constraint is expressed as:

$$U_m = 1 \tag{17}$$

The local dimensionless pressure $f\,Re$ drop in the momentum equation is calculated in terms of the Fanning factor, which is defined as:

$$f = \frac{\left(-\frac{\partial p}{\partial x}\right) D_h}{2\rho u_m^2} \tag{18}$$

and the Nusselt number is defined as:

$$Nu = \frac{hD_h}{k_f} \tag{19}$$

Where k_f is the thermal conductivity of the fluid and h the peripherally averaged heat transfer coefficient, which is given by:

$$h = \frac{k_f u_m (D_h/4)}{\alpha_f (T_w - T_m)} \frac{\mathrm{d}T_m}{\mathrm{d}x} \tag{20}$$

Accordingly, the expression for the peripherally averaged Nusselt number is reduced to:

$$Nu = -\frac{1}{4\theta_m} \tag{21}$$

Numerical procedure

The forgoing equations together with the given boundary conditions are solved numerically using the control volume formulation outlined by Patankar (1980), which ensure conservation of momentum and energy over each control volume, and, thus across the fluid/porous insert interface as well. The sudden change in the diffusion coefficients, at the fluid/porous insert interface, is handled by use of the harmonic mean to ensure conservation and uniqueness of mass and heat fluxes. The nonlinear terms in the momentum equation arising from the non-Newtonian fluid and the presence of the porous material are treated as source terms and linearized as described by Patankar (1980).

The algebraic equations are solved using the Gauss-Seidel method. First, the velocity field is determined since it is independent of the temperature field, thus the velocity values so obtained are used to solve for temperature field. The convergence of computations is assumed to be obtained when the error on the mean velocity is satisfied to three significant digits and when the absolute value of the relative error for the velocity and the temperature at each grid point is found to be less than 10^{-5}.

Necessary mesh size required to reach a sufficient accuracy is found by trial and errors. It is established that a uniform 50 by 50 grid yielded results less than 1 percent from those obtained using a 60 by 60 uniform grid.

The accuracy and validity of the numerical model is verified by comparing results from the present work, for limit cases, with corresponding results reported in the literature. As it is shown in Table I, results of dimensionless pressure drop ($f\,Re$) are compared with those given by the correlation reported by Hartnett and Kostic (1989):

$$f\,Re = \frac{7.4942\left(\frac{1.733}{n} + 5.8606\right)^{n}}{4} \tag{22}$$

The numerical predictions of the peripherally averaged Nusselt number for the case of forced convection in nonporous duct ($Da \to \infty$) and the results reported by Syrjälä (1996) are given in Table II for different aspect ratios and power-law indices. It appears from Table II that the agreement is very good for $n = 1$ and 1.5, while the difference is not significant for $n = 0.5$, the relative error being less than 1.3 percent.

n	0.5	1	1.5
Correlation (22)	5.7217	14.227	34.82
Present study	6.0092	14.203	33.24

Table I. Comparison of *fRe* results with Hartnett and Kostic (1989)

		n			
	r	0.1	0.2	0.5	1
Present study	0.5	7.1043	5.9806	4.4065	3.9575
Syrjälä (1996)	0.5	7.1066	5.9789	4.3979	3.9066
Present study	1	6.7924	5.7390	4.1235	3.6099
Syrjälä (1996)	1	6.7850	5.7377	4.1233	3.6080
Present study	1.5	6.6373	5.6357	4.0272	3.4879
Syrjälä (1996)	1.5	6.6510	5.6510	4.0387	3.4938

Table II. Comparison of *Nu* results with Syrjälä (1996)

Results and discussion

Hydrodynamic results are presented in terms of velocity profiles and dimensionless pressure drop, while heat transfer results are given in terms of peripherally averaged Nusselt number and temperature field. The effect of the porous layer thickness, varying from 0 to 100 percent of the gap, is analyzed. The effect of the permeability is taken into account by varying the Darcy number over a wide range. The rheological aspect is considered by taking different values of the power-law index. The following results are presented for the case where the inertia effects are neglected.

Hydrodynamic results

The use of a porous medium as a technique for heat transfer enhancement results in a penalty due to the increased pressure drop. This is an important factor to be considered, because in industrial applications, this added pressure drop is the price to be paid the heat transfer enhancement by using a porous matter.

The variation of the dimensionless fRe pressure drop in duct is shown in Figure 2, which shows the combined effects of the porous layer thickness and the power-law index n on the axial pressure. It is shown that the pressure drop increases significantly with the increase of n in the Darcy regime. Indeed, the obtained pressure drop, with shear-thinning fluids and a completely porous duct, is lower than that obtained with a shear-thickening fluid approximately of 1,000 times with an entirely porous duct and approximately of 10 times with a duct filled in 80 percent of the porous material. As Da increases, the difference in the pressure loss exhibited by the considered fluids, for fixed e, diminishes until to reach the non-porous case. This result is in agreement with that reported by Hartnett and Kostic (1989). Besides, for relatively weak thicknesses, as shown in Figure 2(a), the pressure loss remains practically constant as long as Da is approximately lower than 10^{-4} for shear-thinning fluids and Da lower than 10^{-3} for shear-thickening fluids, thus the variations of the porous material permeability does not affect the pressure loss. This phenomenon is explained by the fact that for these Da values, the fluid does not virtually cross the duct. For thicker porous layers, it is interesting to note, as shown in Figure 2(b), that the pressure drop for the 90 percent-porous-material-filled duct is the same as the one for the fully porous duct above a critical value of $Da \cong 10^{-4}$. For the 80 percent-porous-material-filled duct, the critical value Da increases 10^{-3}.

Figure 3 shows how the flow field is altered when a porous material fills the duct. The geometry considered has two symmetry planes, so only the velocity profile about $Y = 0.5$ is analyzed here. As it is shown in Figure 3(a), at low permeabilities, virtually all the flow rate is concentrated in the center of the duct characterized by very a high value of the centerline velocity because of the high resistance encountered by the flow due to the porous insert. When Da increases, more flow rate penetrates the porous layer, improving the convective activity in the porous layer. Similar observations are made with a porous layer thickness equal to 0.2 from Figure 3(b). However, the variations of the velocity at the interface are less pronounced.

Figure 4 shows the effects of the porous layer thickness on the velocity profile for a fixed permeability. As long as the porous layer occupies less than 60 percent of the duct section, the velocity profiles are flattened, which basically corresponds to the Darcian regime. It is also showed that the velocity gradient decrease when the porous material thickness decreases from $e = 0.5$ to 0.4. Such flow behavior will have a significant impact on heat transfer as will be shown later.

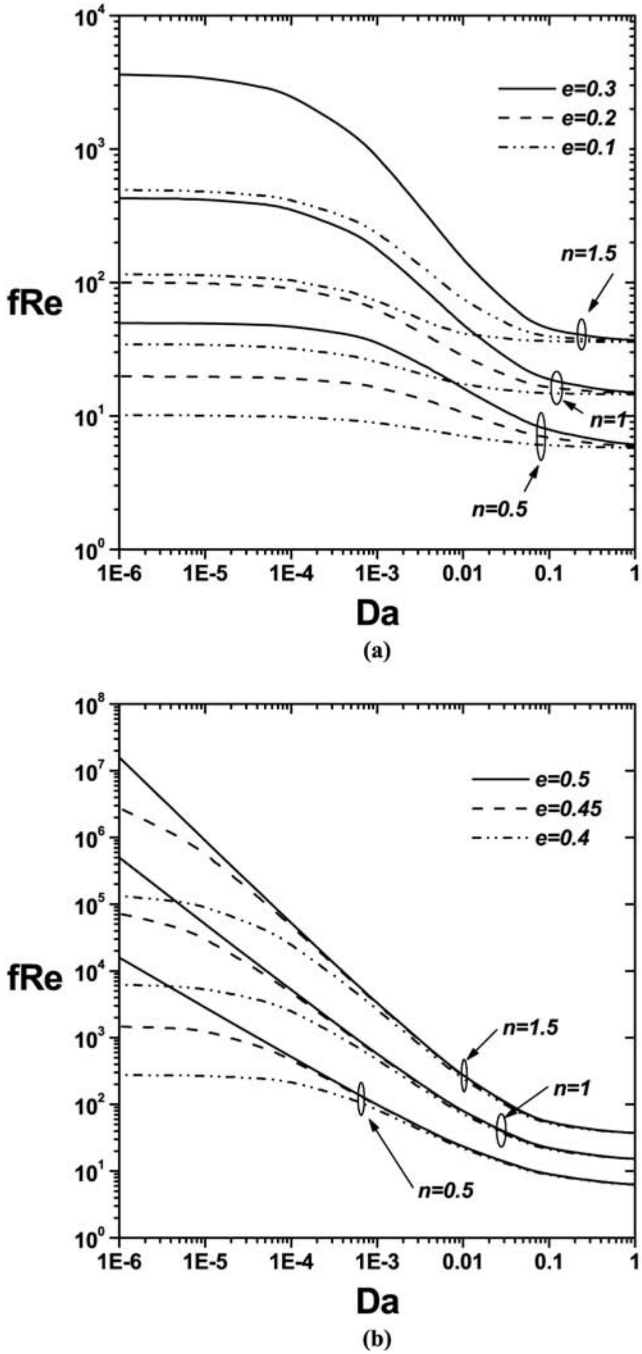

Figure 2.
Variations of the dimensionless pressure drop with the Darcy number and the porous layer thickness; (a) $e = 0.1$, 0.2, 0.3; (b) $e = 0.4$, 0.45, 0.5

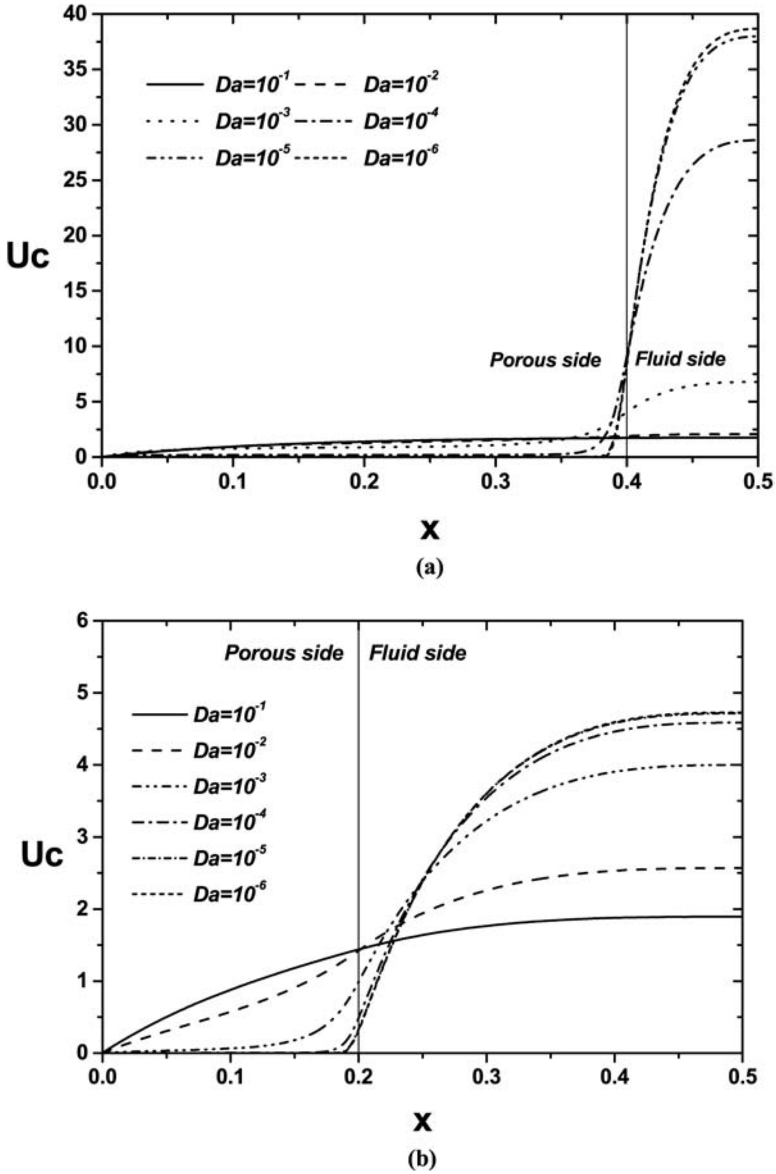

Figure 3.
Effects of Darcy number on the velocity profiles at the plane of symmetry for $n = 0.5$; (a) $e = 0.4$; (b) $e = 0.2$

Heat transfer results

Figure 5(a) and (b) show the evolution of Nu against the Darcy number Da for different values of the porous layer thickness, and for $n = 0.5$ and 1.5. It appears that the thermal behavior shown from these graphs is globally the same for shear-thinning and shear-thickening-fluids. Let's analyze Figure 5(a), which concerns shear-thinning fluids ($n = 0.5$). Three different behaviors of Nu are shown in this graph. For a porous layer

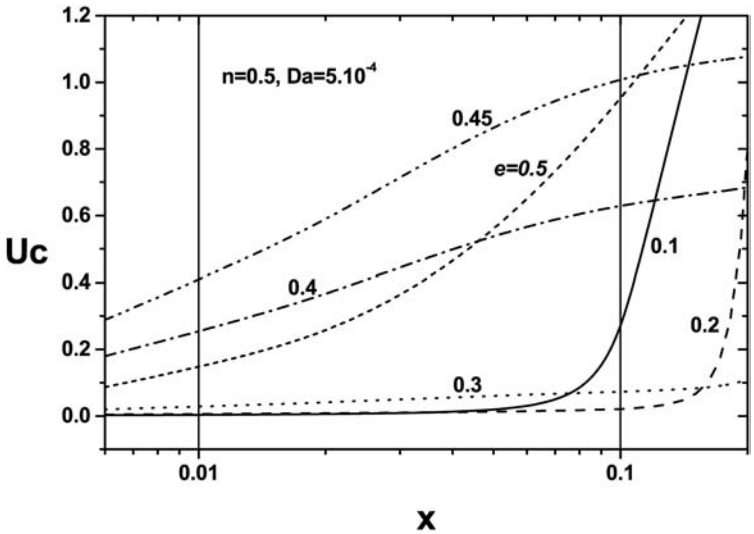

Figure 4.
Effects of porous layer thickness on centerline velocity for a fixed permeability

thickness of 0.1, 0.2 and 0.3, the peripherally averaged Nusselt number increases with permeability until it reaches an asymptotic value, which corresponds to the non-porous case. Moreover, for low permeabilities, until approximately $Da = 10^{-4}$, the increase of the porous layer thickness leads to heat transfer fall. This is due, first to the increase of the hydrodynamic resistance to the flow induced by the presence of the porous medium which makes that the majority of the flow rate passes through the non-porous region, and second, to the augmentation of the thermal resistance which makes the heat flux decrease. Thus, it can be concluded that the heat transfer occurs only through conduction for the first case.

The second behavior is the one observed for thicknesses of the porous layer from to 0.4 to 0.45. For this case, the *Nu* increases when the porous layer permeability increases up to a critical value ($Da = 10^{-3}$ for $e = 0.45$) beyond which, *Nu* decreases. Unlike the precedent case, comparatively to the non-porous duct, heat transfer improvement can be obtained from a critical value of *Da*. This enhancement is better for thicker layers.

When the porous layer fills completely the duct ($e = 0.5$), a heat transfer enhancement is obtained for any permeability; although, this enhancement decreases with the increase of *Da*. This behavior can be explained by the fact that at low permeabilities, very high velocity gradients occur near the duct wall. This results in strong convective effects, and leads to an increase of heat transfer. As long as the Darcy number is increased, the velocity gradients at the duct walls decrease, which results in the boundary layer thickness increase, and thus a *Nu* decrease.

In addition, one cannote some critical points beyond which the *Nu* evolution in terms of *Da* is reversed. For example, at $Da = 5 \times 10^{-4}$, heat transfer obtained with a porous layer of thickness about 0.4 becomes greater than heat transfer obtained with a 0.3 thickness porous layer. The physical explanation is that, at this permeability, the velocity gradient created in the vicinity of the duct walls is more important with a porous insert thickness equal to 0.4 according to Figure 4. For shear-thickening fluids similar observations are noted.

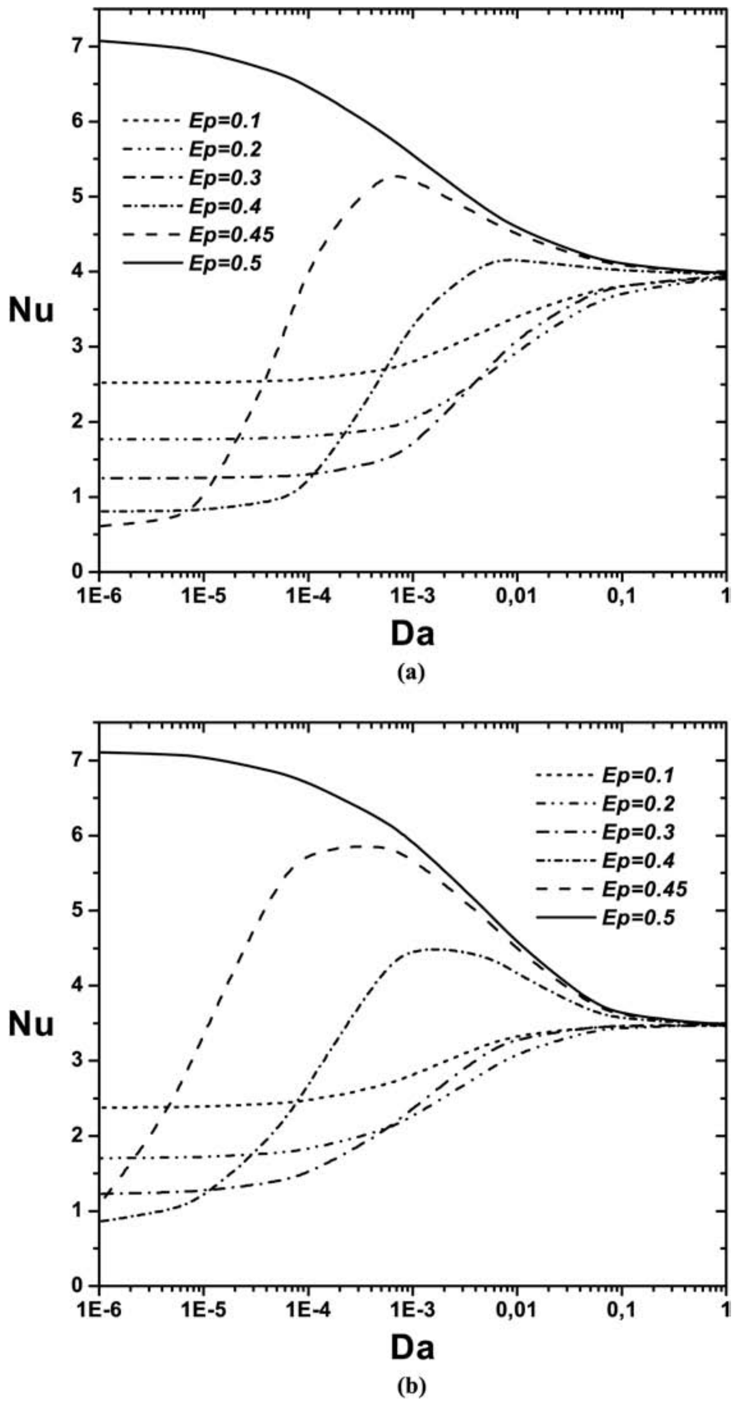

Figure 5.
Variation of the peripherally averaged Nusselt number against *Da* for different porous layer thicknesses; (a) $n = 0.5$; (b) $n = 1.5$

The effect of the power-law index on the heat transfer is shown in Figure 6. When the porous substrate fills to 40 percent of the duct section, the rheological properties of the fluid influence slightly the heat transfer, except at high permeability where shear-thickening fluids exhibit the highest Nusselt number. For a porous layer thickness equal to 0.4, *Nu* remains virtually constant in the Darcian regime, while as the Darcy number increases and the power-law index is increased, *Nu* increases, with a relatively constant difference, up to a critical *Da* value for which heat transfer is maximal; above this value the difference in *Nu* diminishes. While shear-thickening fluids exhibit the highest *Nu* above another critical *Da*, approximately equal to 10^{-2}.

Variations of the Nusselt number as a function of the porous layer thickness are shown in Figure 7. The conjugated effects of the power-law index and the permeability are also analyzed for a low conductivity porous insert ($R_k = 1$). For a given *Da*, the Nusselt number decreases with the increase of the porous layer thickness, until a critical value, above which *Nu* increases. Similar results were reported by Chikh *et al.* (1995a) for a partially annular porous duct. However, in their results, no heat transfer enhancement was obtained, while in this study, a heat transfer improvement is realized, in spite of the low conductivity, beyond a critical thickness which is depending on the permeability and the power-law index. The impact on heat transfer enhancement of the power-law index is more clearly illustrated in this figure. Indeed, when $Da = 10^{-1}$, shear-thinning fluids exhibit a more important heat transfer than shear-thickening fluids whatever the porous layer thickness. As *Da* decreases, a critical thickness appears, for which both shear-thinning and shear-thickening fluids give the same heat transfer rate. Below this value, the obtained heat transfer is better with shear-thinning fluids while above this value the inverse behavior is observed.

The effects of porous material conductivity on heat transfer are analyzed. Several values are considered, from 1 to 100, as shown in Figure 8 for shear-thinning fluids. Beyond a certain thermal conductivity ratio, which depends on the permeability, heat transfer enhancement can be achieved significantly. However, at $Da = 10^{-4}$ using

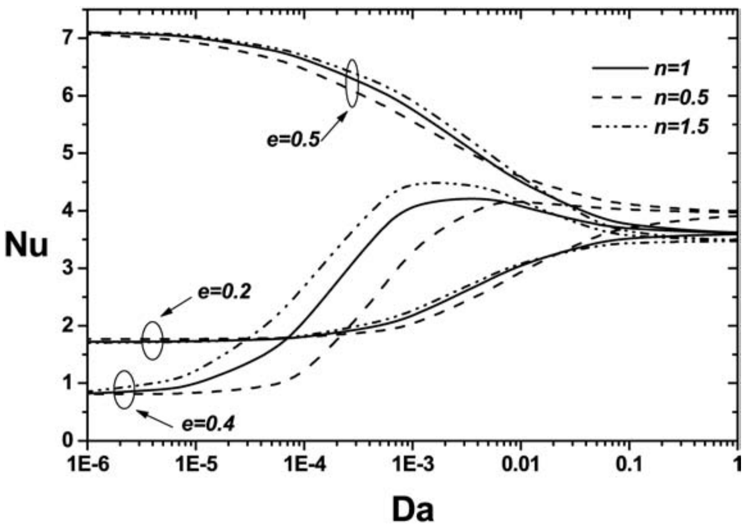

Figure 6.
Effects of the power-law index on the heat transfer versus *Da* for different thicknesses of the porous layer

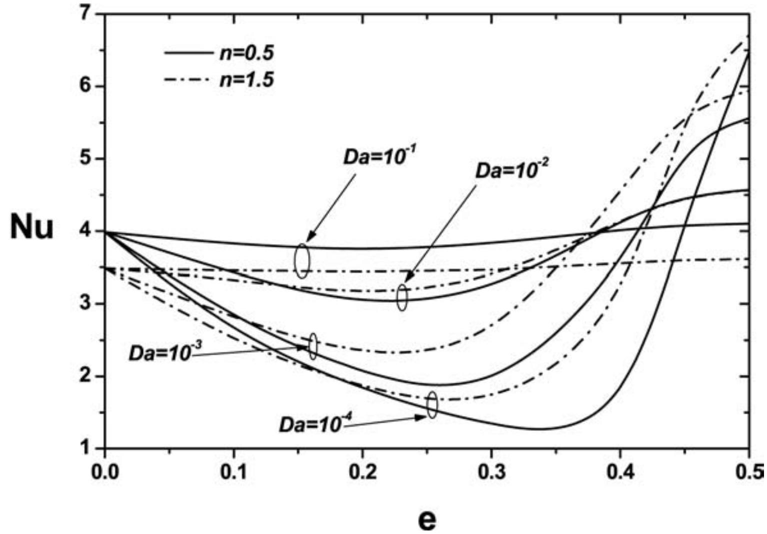

Figure 7.
Variations of Nusselt number as a function of the porous layer thickness

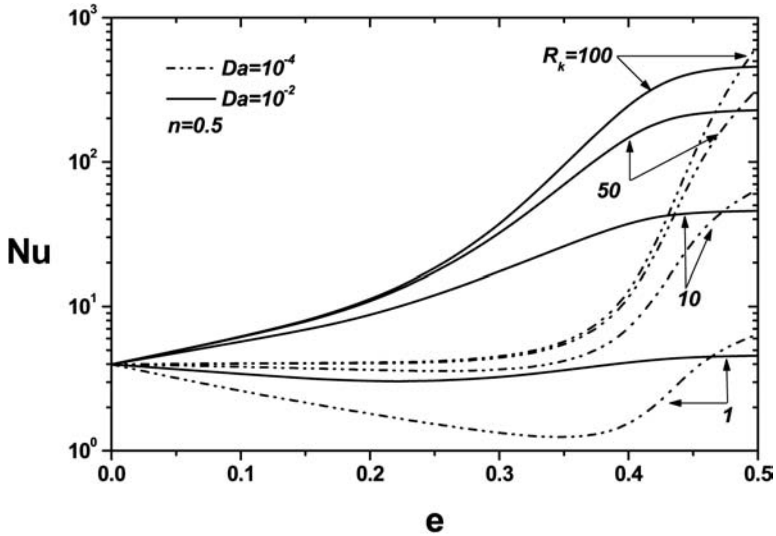

Figure 8.
The effects of the porous material conductivity and e on heat transfer

more conductive porous material does not improve the heat transfer as long as the thickness is less than 0.4. As it was explained previously, this is caused by the fact that not enough fluid crosses the porous layer. For low permeability, when the porous layer occupies close to 100 percent of the duct section, the *Nu* becomes higher, in accordance with the results shown in Figure 4, where high velocity gradients appear at these thicknesses.

The variation of the peripherally averaged Nusselt number against the conductivity ratio is shown in Figure 9, which displays the combined effects of the power-law index

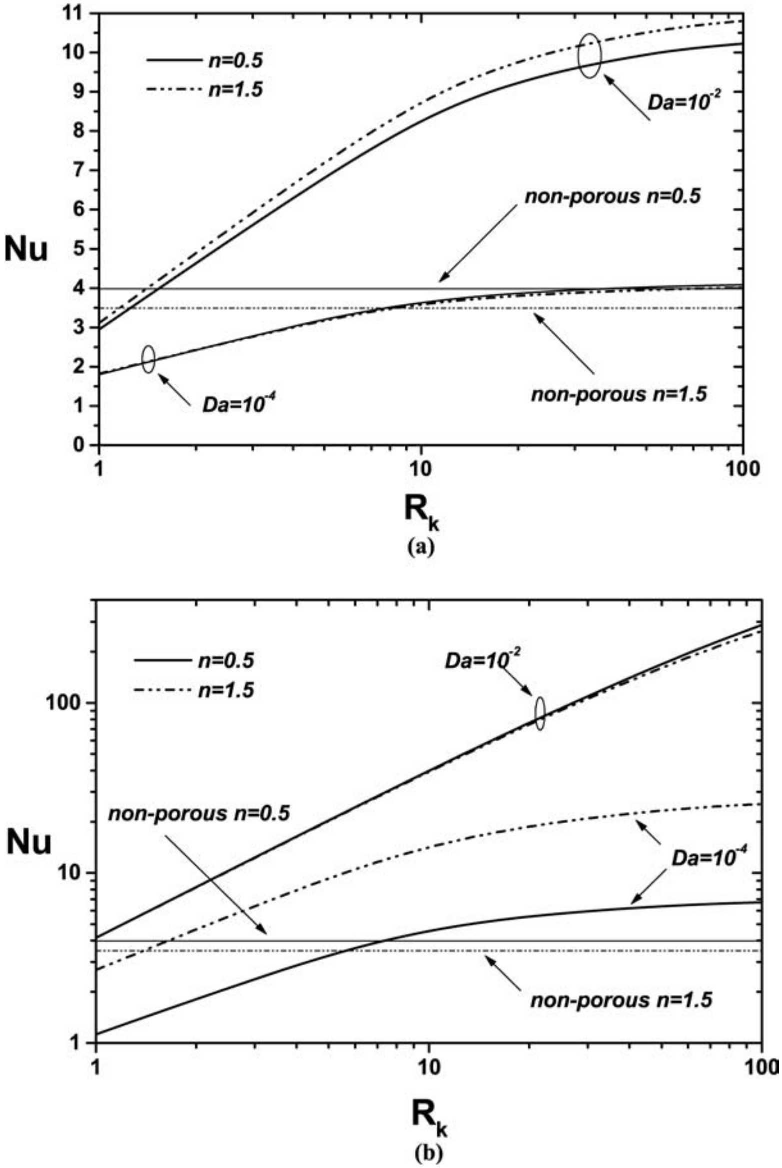

Figure 9.
The variations of the peripherally averaged Nusselt number against the conductivity ratio; (a) $e = 0.2$; (b) $e = 0.4$

and the permeability. Figure 9(a) shows that heat transfer enhancement is obtained, compared to the non-porous case, beyond a certain value of the thermal conductivity ratio. This enhancement is significant at high permeability for both for shear-thickening and shear-thinning fluids. When the permeability of the porous substrate is low, the heat transfer enhancement is weak for shear-thinning fluids, while for shear-thickening fluids, practically no enhancement is obtained whatever the R_k

value. At low permeability, both shear-thinning and shear-thickening exhibit nearly the same Nu, while at high permeability a slight difference is observed. In the case where the porous material fills the duct to 80 percent, as it is shown in Figure 9(b), heat transfer enhancement is obtained with shear thinning and shear-thickening for any R_k value, at high permeability while at low permeability, enhancement is realized beyond a critical R_k. Contrary to the case where the porous layer fills to 40 percent the duct, the Nu is unaffected by the nature of the fluid at high permeability, while shear-thinning fluids exhibit the highest Nu at low Da. For the both thicknesses considered, at $Da = 10^{-4}$, shear-thinning fluids allow to obtain heat transfer enhancement with less conductive porous matter.

The variations of the averaged Nusselt number with the power law index are plotted in Figure 10 for two thicknesses of the porous layer and two Darcy numbers. As it is shown, Nu remains virtually constant, except when $e = 0.4$ and $Da = 10^{-4}$. For these conditions, as the power-law index increases, Nu increases. At high permeability, the heat transfer exhibits the highest value when the porous layer is thicker. At low permeability, a critical value of n appears, at which both thicknesses lead to the same heat transfer. Before this value, Nu is lowest when $e = 0.4$ and beyond this value, the inverse behavior is observed.

Conclusion

A numerical study of laminar forced convection in a partially or fully porous square duct saturated with a non-Newtonian power-law fluid was carried out. The flow was assumed to be steady, hydrodynamically and thermally developed. The flow and heat transfer problems were solved numerically using the finite volume method. The effects of the Darcy number, the power-law index, and the thickness of the porous layer on both the flow and heat transfer in the duct were investigated. Among the most important results, it was highlighted that a heat transfer improvement is obtained when the channel is entirely porous, and this enhancement is maximized at low

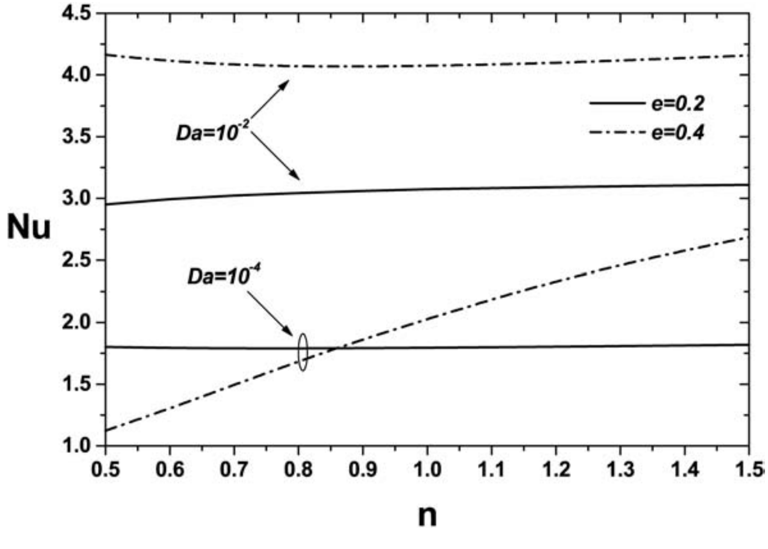

Figure 10.
Variations of Nu as a function of the power-law index

permeability. However, a supplementary pumping effort is necessary, due to the added pressure drop caused by the porous medium. On other hand, depending on the working conditions, heat transfer enhancement could be obtained by filling partially the duct with a porous insert, even if the conductivity ratio is equal to 1. This configuration produces less additional pressure loss. It was also reported that there is a critical thickness depending on the Darcy number, for which the averaged Nusselt number is minimized. Thus, under these specific conditions, according to the required objectives, the porous insert can be used either for insulation or for heat transfer enhancement. As it was expected the porous material thermal conductivity has a strong effect on the heat transfer. It was shown that at high permeability the heat transfer enhancement increases with an increasing conductivity ratio and the porous layer thickness. While at low permeability, using more conductive porous matter does not improve further the heat transfer enhancement as long as the porous layer thickness is less than a certain critical value. Concerning the impact of the rheological properties of the fluids on the heat transfer, it was demonstrated that both shear-thinning and shear-thickening fluids allow obtaining the highest Nusselt number depending on the properties of the porous insert. The added pressure drop was found to be significantly more important with shear-thickening fluids in comparison to the Newtonian and the shear-thinning fluids either when the porous matter fills partially or entirely the duct. One can conclude that favorable and unfavorable conditions for heat transfer enhancement exist according to the properties, the thickness of the porous insert and the power-law index.

This study appears to be the first devoted to forced convection in a porous square duct saturated with a power-law fluid. Clearly, further studies are needed to investigate the influence of others parameters such as aspect ratio, the non-Darcian effects, thermal dispersion, etc.... Also, in light of the results obtained supplementary developments are required to determine the optimum conditions for maximizing the heat transfer enhancement and to maintain the pressure drop to acceptable values.

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